Quiz 4A, Business Calculus

Fall 2014 - Dr. Graham-Squire

8:18
8:27
9
↳ dt 25 min.

Name: Key

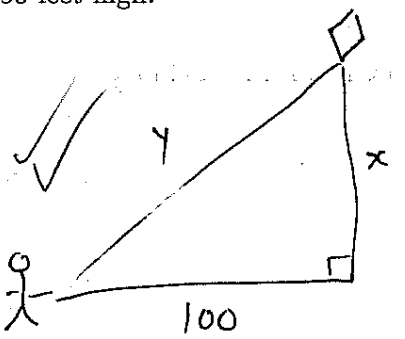
Round to nearest 0.01

1. (4 points) Rishi is flying his kite outside. Unfortunately, the wind has blown his kite up against a building, and the kite is sliding up the side of the building as Rishi lets the kite string out of the spool. Suppose that Rishi is standing 100 feet from the building (over flat ground), and he is releasing string from the spool at a rate of 3 feet per second.

   (a) Draw a diagram of the situation, placing variables appropriately on the diagram.

   (b) Use calculus to determine how fast the kite is sliding up the side of the building when the kite is 250 feet high.

(a)



Know: $\frac{dy}{dt} = 3$ ✓

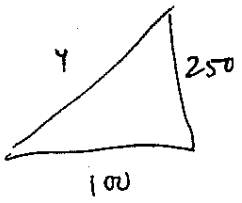
Want: $\frac{dx}{dt}$ when $x = 250$

$\frac{d}{dt}\left(100^2 + x^2 = y^2 \right)$ ✓✓

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ ✓

$2(250)\frac{dx}{dt} = 2(269.29)(3)$ ✓

$\frac{dx}{dt} = \frac{6(269.29)}{500} = 3.23$ ft/sec ✓



$y^2 = 250^2 + 100^2$

$y = \sqrt{62500 + 10000}$

$y = 269.29$

2. (6 points) Let $f(x) = \dfrac{x^3 - 6x^2}{x+2}$. Then its first and second derivatives are

$$f'(x) = \frac{2x(x^2-12)}{(x+2)^2} \qquad f''(x) = \frac{2(x^3+6x^2-12x-24)}{(x+2)^3}$$

You DO NOT need to calculate these derivatives yourself. I did them for you, just trust me that they are true. Answer the following questions about $f(x)$. You can use the graphing function on your calculator to check your work, but you must show your work using the calculus techniques we have used in order to receive full points.

(a) Find the interval(s) on which f is increasing. If there are none, write "No intervals".

(b) Find the x-value(s) at which f has a local maximum. If there are none, write "No maximums".

(c) Calculate the absolute extrema (that is, absolute maximum and absolute minimum) of f on the interval [-0.5, 7].

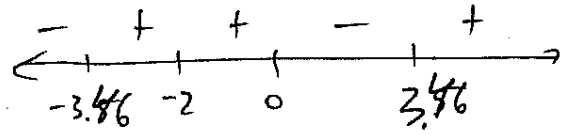
(d) Find one x-value where f is concave up. Explain how you know (it is not enough to say you saw it on the graph).

(a) $2x = 0 \Rightarrow x = 0$

$x^2 - 12 = 0 \Rightarrow x = \pm\sqrt{12}$

$x = \pm \approx 3.46$

$x + 2 = 0 \Rightarrow x = -2$



test $f'(-4) = -$

$f'(-3) = +$

$f'(-1) = +$

$f'(1) = -$

$f'(4) = +$

increasing on $(-3.46, -2), (-2, 0), (3.46, \infty)$

(b)  local max @ $x = 0$

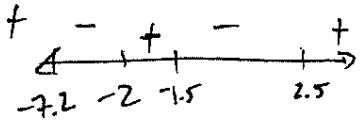
(c) $f(-0.5) = -1.083$

$f(0) = 0$

$f(\sqrt{12}) = -5.569$ ← abs. min

$f(7) = 5.44$ ← abs. max

(d) Want $f''(x)$ to be positive, would occur at a minimum like $x = \sqrt{12}$. Check $f''(3) = \dfrac{2(27+54-36-24)}{5^3} = \dfrac{42}{53}$ is positive.



So $x = 3$ works

or anything in intervals $(-\infty, -7.2), (-2, -1.5), (2.5, \infty)$